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Stability Analyses of Car-following Model Based on The Root Locus Method

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Abstract: Based on the different sensitivities as drivers to response for the space headway or the velocity difference between the following vehicle and the leading vehicle, this study proposes a new car-following (CF) model that considers two different time delays in sensing space headway and velocity. The stability analysis of the proposed CF model is performed using the root locus method to obtain the stability condition. This approach is able to analyse all aspects of the dynamics: long waves and short wave behaviours, phase velocities and stability features. Simulation-based numerical experiments are conducted. Results show that the stable region of the proposed model varies with the difference between two different time delays.
Keywords: car-following model, time delays, stability analysis, the root locus method.

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1. INTRODUCTION

The Traffic problems, e.g., traffic congestion, traffic pollution, and traffic management [1-3], have been received considerable attention in academic and industrial circles. To address the above mentioned issues, one effective way is to model the traffic movement to capture the mechanisms behind the phenomena of vehicular traffic flow [5-7]. Consequently, various traffic flow models have been proposed from the microscopic and macroscopic viewpoints. This study focuses on the microscopic traffic flow model. From the microscopic model perspective, optimal velocity (OV) model [8] is popular in the field of traffic flow. The OV model introduced the concept of optimal velocity into the modeling traffic flow based on the assumption that the driver of vehicle will seek a safe velocity determined by the distance from the leading vehicle in car-following process. Since then, numerous OV-based CF models have been proposed, such as full velocity difference (FVD) model [9], multiple headway velocity and acceleration difference (MHVAD) model [10] and so on (see [11-13] for a review). The aforementioned CF models can be further categorized into lane-discipline-based models, where the models restrict the limitation to the assumption that vehicles follow the lane discipline and travel in the middle of the lane. However, these models may not be valid in the scenario where lanes may not be clearly demarcated on a road though multiple vehicles can travel in parallel [14-18]. Consequently, unlike the lane-discipline-based models, several non-lanediscipline-based models have been proposed with focus on the lateral gap of road in recent years. To address this scenario, Jin et al. [16] proposed a non-lane-based full velocity difference CF (NLBCF) model to analyze the impact of the unilateral gap on the CF behavior. However, the NLBCF model cannot distinguish the right-side or the left-side lateral gap on a road. Consequently, Li et al. [17] proposed two-sided lateral gap full velocity difference (TSFVD) under non-lane-discipline-based environment. In addition, Li et al. [18] further studied the effects of lateral gaps on the energy consumption for electric vehicle (EV) flow. Li et al. [15] studied the traffic flow behavior considering the effects of visual angle without lane discipline.

Recently, Li et al. [19] proposed a CF model considering the effect of electronic throttle opening angle based on the FVD model. Results from numerical experiments verify the impacts of ET opening angle on traffic flow with respect to the smoothness and stability. However, to the best of our knowledge, no study considers the effects of ET opening angle and lateral gap simultaneously. Hence, there is a research need to study the effects of both ET opening angle and lateral gap on traffic flow behavior.

Broadly speaking, there are two types of stability analysis: linear stability analysis and nonlinear stability analysis. Linear stability analysis focuses on stability characteristics of a system under the influence of a small perturbation (in this case nonlinear system can be linearized to deal with the small perturbation), while nonlinear stability analysis on stability characteristics of a system under the influence of a large perturbation. For two main reasons, linear stability analysis has been the theme of the majority of the literature in traffic flow theories where nonlinear CF system is often linearized around the equilibrium point: i) for road traffic, disturbances experienced by road users are often small; and ii) nonlinear stability analysis is much more complicated than linear stability analysis. Thus, linear stability analysis is the focus of this paper, too. Before any further discussion, some confusion on terminologies used in the literature needs to be clarified. The fact that stability analysis is an important topic, and thus has been investigated in different disciplines (e.g., numerical analysis, control theory, dynamic systems, and of course traffic flow modelling), has caused confusions in the literature in terms of terminologies. In dynamic systems, only local stability is studied, which is categorized as Lyapunov stability (any sufficiently small initial perturbation always remains small) and asymptotic stability (any sufficiently small initial perturbation tends to zero as time approaches infinity). In traffic flow modelling, two types of stability have also been investigated but defined differently: Local stability is to investigate the stability of a single vehicle's movement over time under the influence of a small perturbation that is often originated from the leading vehicle's movement (It is locally stable if the perturbation diminishes over time, which corresponds to asymptotic stability in dynamic systems), while asymptotic stability focuses on the stability of a platoon of vehicles over space under the influence of a small perturbation that is originated from the first vehicle of the platoon (it is asymptotically stable if the perturbation diminishes over space/vehicles). Some researchers in traffic flow modelling also call these two types of stability analysis as single vehicle stability (or platoon stability/plant stability), and stability over vehicles (or string stability), respectively. This paper investigates the linear stability of traffic flow dynamics using the root locus method. It is clearly seen that the method allows all the aspects of the dynamics to be analysed: long and short wave behaviours can be studied, phase velocities and stability features can be exhibited. In this paper the method is applied to cooperative and non-cooperative microscopic CF models, however, the same approach can be applied to other modelling levels as well. In addition, this method has the advantage to bring out specific collective behaviours and the way to pass from one cooperative regime to the other.

In the car-following models, many parameters appear to describe the physical conditions of roads, mechanical properties of vehicles, states of drivers, traffic laws etc [20]. Among these parameters, time delay is a critical one which has been recognized in the traffic studies since 1958. Bando et al. [8] demonstrated that in the OV model delay times of vehicles coming from the dynamical equation of motion of the OV model explain the order of delay times observed in actual traffic flows without introducing explicit delay times. They found that in most of the situation for which an effective delay time can be made a reasonable definition. Davis [21] posed a traffic model with a delay time due to driver's reaction time. It consists of a system of delay differential equations and is a variation of the OV model.

It is known that time delays mainly originated from the time needed by drivers in sensing stimulus, making decision and performing control actions against real-time variations in the dynamical traffic flow. Some papers study only the driver's time delay in sensing space headway. In general, the time delays in sensing space headway and in sensing velocity are different [22]. But for the sake of simplicity, the different kinds of time delays are considered to be equal to each other or to be zero. Yu et al. [23] proposed a new car-following model by considering two different time delays, while not illustrate the effectiveness of the time delays in complex traffic flow models. In this study, we proposed a new traffic flow model considering the effectiveness of different time delays. In addition, the stability conditions are analyzed under different model parameters and different time delays.

The remainder of paper is organized as follows. A new car-following model is proposed by taking into account two different time delays in sensing space headway and velocity in Section 2. Section 3 analyzes the stability of the proposed car-following model. Section 4 performs the numerical experiments and comparisons. The final section concludes this study. Inclination of cylinder makes an axial displacement of the solid bed, which moves towards the discharge end. The rotating cylinder acts simultaneously as a conveying device and stirrer by the use of internal fins which helps to mix and rotate the material in radial direction. Inclination angle of the cylinder Kiln control is one of the most vital parts and the kiln is very sensitive for operation. Control of the kiln during its operation the assemblage of various components and process parameters is essential one in the rapid fast developing environment [1].

2. METHODOLOGY

2.1 Model derivation

The dynamic equations of the electronic throttle (ET) angle to vehicle velocity are as follows [8]:

$$a_n(t) = -b(v_n(t) - v_0) + c\bar{\theta}_n + d_n \tag{1}$$

where v_0 is the steady-state vehicle velocity for a throttle input θ_0 and $\bar{\theta}_n = \theta_n - \theta_0$ as the throttle deviation from θ_0; b and c are positive parameters; d_n is the perturbation to capture the effect of un-modeled dynamics; and $v_n(t)$, $a_n(t)$ are the velocity and acceleration of the vehicle n at time t, respectively.

The dynamical equation of the nth vehicle of the T-FVD model [8] is given by

$$a_n(t) = k[V(\Delta x_n(t)) - v_n(t)] + \lambda \Delta v_n(t) + \kappa \Delta \theta_n \tag{2}$$

where $a_n(t)$ is the acceleration of the nth vehicle at time t, $v_n(t)$ is the velocity of the nth vehicle at time t, $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ is the space headway between two successive vehicles of the $n+1$th and the nth vehicles, $\Delta v_n(t) = v_{n+1}(t) - v_n(t)$ is the velocity difference (i.e., the relative velocity), $\Delta\theta_n = \theta_{n+1} - \theta_n$ represents the ET opening angle difference , $V(\Delta x_n(t))$ is the optimal velocity function, k is the sensitivity and λ is the coefficient of the relative velocity, κ is the sensitivity coefficient. In this case each driver's response to the stimulus from the vehicle ahead of the driver is instantaneous. The current paper is concerned with an extended T-FVD model which contains two different time delays, where one in sensing space headway and the other in sensing relative velocity. The acceleration of the nth vehicle is given by a delay differential equation

$$a_n(t) = k[V(\Delta x_n(t-\tau_1)) - v_n(t-\tau_2)] + \lambda \Delta v_n(t-\tau_2) + \kappa \Delta \theta_n \tag{3}$$

Where $\tau_1 > 0$, $\tau_2 > 0$ are introduced to represent the time delays of in sensing space headway and in relative velocity. For simplicity, τ_1 and τ_2 are assumed to be the same for all drivers. When $\tau_1 = \tau_2 = 0$, the proposed time-delays-based (T-FVD-D) model turns to the T-FVD model [8].

The optimal velocity function V is given

$$V(\Delta x(t)) = \frac{v_{\max}}{2}[\tanh(\Delta x(t) - h_c) + \tanh(h_c)] \tag{4}$$

Where h_c the safety is space headway and $v_{\max}$ is the maximum velocity. The optimal velocity function is a monotonically increasing function of the space headway and has an upper bound, i.e., the maximal velocity. When the space headway is less than the safety distance, the vehicle reduces its velocity to prevent from crashing into the preceding vehicle. On the other hand, if the space headway is larger than the safety distance, it increases toward the maximal velocity.

2.2 Stabilty Analysis

In this section, a thorough stability analysis is given to study the effect of the two different time delays on the stability condition of the T-FVD-D model. To obtain the stability condition of the T-FVD-D model, we perform the stability analysis using the root locus method. The stability of the T-FVD-D model is analyzed under the following assumption.

Assumption 1 The initial state of the vehicular traffic flow is steady, and all vehicles in the traffic flow move with the identical space headway and the optimal velocity. Following the Assumption 1, the position solution to the steady vehicular traffic flow is

$$x_n^{(0)}(t) = hn + V(h)t \tag{5}$$

Where h the steady is space headway and $V(h)$ is the optimal velocity in uniform traffic flow. $x_n^{(0)}(t)$ is the position of the nth vehicle in steady state.

Adding a small disturbance $y_n(t)$ to the steady-state solution, i.e. ,

$$y_n(t) = x_n(t) - x_n^{(0)}(t) \tag{6}$$

Note that $\Delta x_n = y_n + h$, $v_n = \dot{y}_n + V(h)$, $a_n(t) = \ddot{y}_n(t)$ and substituting formula (6) into formula (2), the resulting equation is as follows:

$$\ddot{y}_n(t) = k[V'(h)\Delta y_n(t) - \dot{y}_n(t)] + \lambda \Delta \dot{y}_n(t) + \kappa \Delta \theta_n \tag{7}$$

Where $V'(h)$ is the derivative of optimal velocity function $V(\Delta x_n(t))$ at $\Delta x_n(t) = h$ and $\Delta y_n(t) = y_{n+1}(t) - y_n(t)$.

According to formula (1), the opening angle of ET, θ_n, can be rewritten as follows:

$$\theta_n = \frac{1}{c}(\ddot{y}_n + b\dot{y}_n - d_n) + \theta_0 \tag{8}$$

Hence

$$\Delta\theta_n(t) = \frac{1}{c}(\Delta\ddot{y}_n(t) + b\dot{y}_n(t)) \tag{9}$$

Substituting formula (9) into formula (7), the equation is as follows:

$$\ddot{y}_n(t) = k[V'(h)\Delta y_n - \dot{y}_n] + \lambda\Delta\dot{y}_n + \frac{\kappa}{c}(\Delta\ddot{y}_n + b\Delta\dot{y}_n) \tag{10}$$

when contains two different time delays one in sensing space headway and the other in sensing relative velocity in formula (10), the linearized equation for $y_n(t)$ is obtained from the T-FVD-D model

$$\ddot{y}_n(t) = k[V'(h)\Delta y_n(t-\tau_1) - \dot{y}_n(t-\tau_2)] + \lambda\Delta\dot{y}_n(t-\tau_2) + \frac{\kappa}{c}(\Delta\ddot{y}_n(t) + b\Delta\dot{y}_n(t-\tau_2)) \tag{11}$$

Set $y_n(t)$ in the Fourier models, i.e., $y_n(t) = A\exp(\mathrm{i}\alpha n + zt)$, the following equation of z is obtained from formula (11)

$$z^2 = kV'(e^{\mathrm{i}\alpha} - 1)e^{-z\tau_1} - kze^{-z\tau_2} + \lambda z(e^{\mathrm{i}\alpha} - 1)e^{-z\tau_2} + \frac{\kappa}{c}z^2(e^{\mathrm{i}\alpha} - 1) + \frac{\kappa b}{c}z(e^{\mathrm{i}\alpha} - 1)e^{-z\tau_2} \tag{12}$$

Set $e^{z\tau}$ in the Fourier models, i.e., $e^{-z\tau_1} = 1 - z\tau_1 + \frac{1}{2}z^2\tau_1^2$ $e^{-z\tau_2} = 1 - z\tau_2 + \frac{1}{2}z^2\tau_2^2$, the following equation of z is obtained from formula (12)

$$\begin{aligned} &z^2(1 + \tau_2\lambda(e^{\mathrm{i}\alpha} - 1) + \tau_2\frac{\kappa b}{c}(e^{\mathrm{i}\alpha} - 1) - k\tau_2 - \frac{\kappa}{c}(e^{\mathrm{i}\alpha} - 1) - \frac{1}{2}\tau_1 kV'(e^{\mathrm{i}\alpha} - 1)) \\ &+ z(\tau_1 kV'(e^{\mathrm{i}\alpha} - 1) + k - \lambda(e^{\mathrm{i}\alpha} - 1) - \frac{\kappa b}{c}(e^{\mathrm{i}\alpha} - 1)) - kV'(e^{\mathrm{i}\alpha} - 1) = 0 \end{aligned} \tag{13}$$

Based on formula (13), we set $m = (1 + \tau_2\lambda(e^{\mathrm{i}\alpha} - 1) + \tau_2\frac{\kappa b}{c}(e^{\mathrm{i}\alpha} - 1) - k\tau_2 - \frac{\kappa}{c}(e^{\mathrm{i}\alpha} - 1) - \frac{1}{2}\tau_1 kV'(e^{\mathrm{i}\alpha} - 1))$, and $n = (\tau_1 kV'(e^{\mathrm{i}\alpha} - 1) + k - \lambda(e^{\mathrm{i}\alpha} - 1)$, $p = kV'(e^{\mathrm{i}\alpha} - 1)$, It follows from formula (13) that

$$mz^2 + nz - p = 0 \tag{14}$$

Roots of Equation (14) are

$$\begin{cases} z_1 = (-n + \sqrt{n^2 - 4mp})/2m \\ z_2 = (-n - \sqrt{n^2 - 4mp})/2m \end{cases} \tag{15}$$

The table 1 shows the arrangement of rotary kiln with accessories. Kiln can be used as a rotary dryer to remove water and moisture content from solid substances by introducing hot gases into a drying chamber. Kiln shell should be structurally strong with non-conductor lining and designed to withstand high temperature and prevent the thermal losses of the kiln. Construction and position alignment of the kiln is very important for all the process. In thermal processing of residual materials with a various origin and predominantly for fire treatment of hazardous wastes rotary kiln are employed.

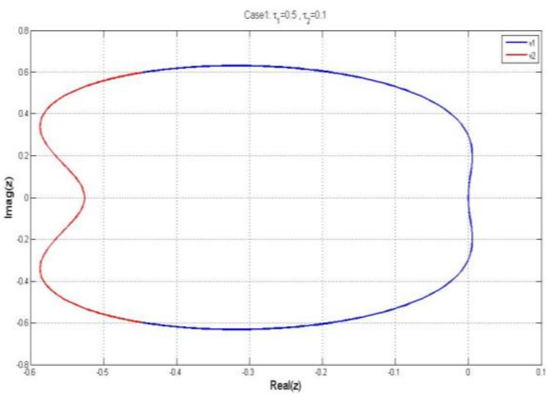
3. NUMERICAL EXPERIMENTS

Based on the foregoing theoretical analyses, the numerical experiments are provided to verify the steady performance of the proposed T-FVD-D model. Without loss of generality, we compare the proposed T-FVD-D model with the existing T-FVD model according to the stable region. Starting from the simulation, the values of parameters related to the proposed model are set in Table1.

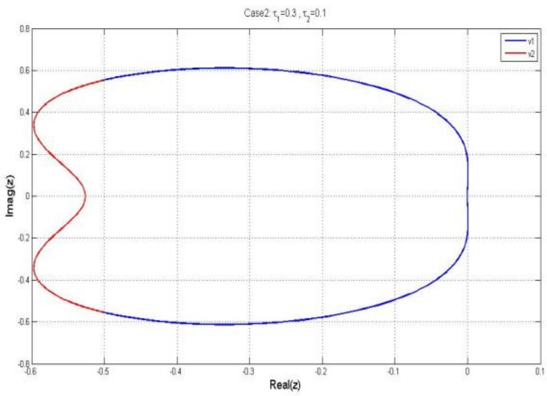
Table 1 Values of the relevant parameters

element	v_{max}	κ	b	c	λ
content	3.0	0.1	0.8	0.27	0.3

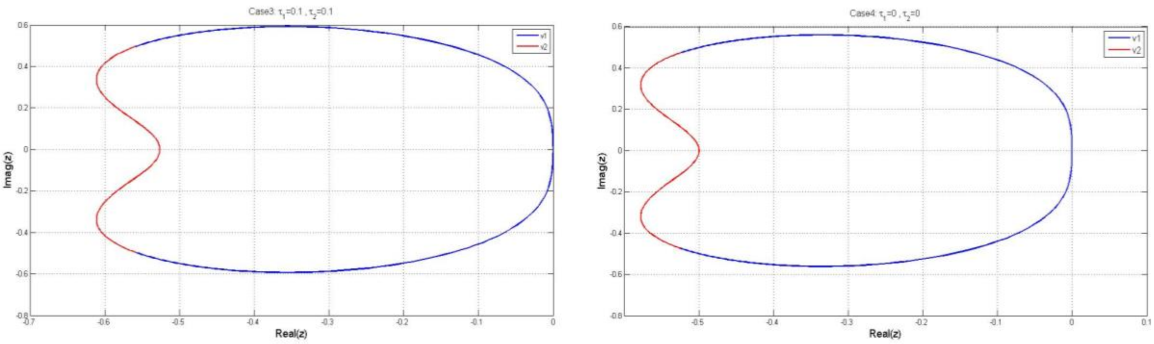
We analyze the stability region of the T-FVD-D model by the root locus method. The root locus is the path of the roots traced out in the complex plane as evolves. If all the roots locates at the left half plane (The complex plane is separated into two parts: the left half plane, corresponding to negative real part plus the imaginary axis which is the stability region, and the right half plane which is the instability region), which implies the stability criterion is satisfied. Fig. 1 is an example. As shown in Figs. 1(b) and 1(c),1(d) with a set of parameters that satisfies the stability criterion, all the roots with different locate in the left half plane. Otherwise, some roots locate in the right half of the plane, as shown in Fig. 1(a). Compared with the Fig. 1 (a)-(d), we know that the stable region of the T-FVD-D model would increase with the deceasing of difference between two different time delays. Besides, as can be seen in Fig. 2(a) and 2(b), the stable region is enlarged with the increase in the value of parameter.



(a)



(b)



(c) (d)

Fig. 1 Root locus for different parameters: (a) case 1 $\tau_1 = 0.5$, $\tau_2 = 0.1$; (b)case 2 $\tau_1 = 0.3$, $\tau_2 = 0.1$; (c) Case 3 $\tau_1 = 0.1$, $\tau_2 = 0.1$, (c) case 4 $\tau_1 = 0$, $\tau_2 = 0$

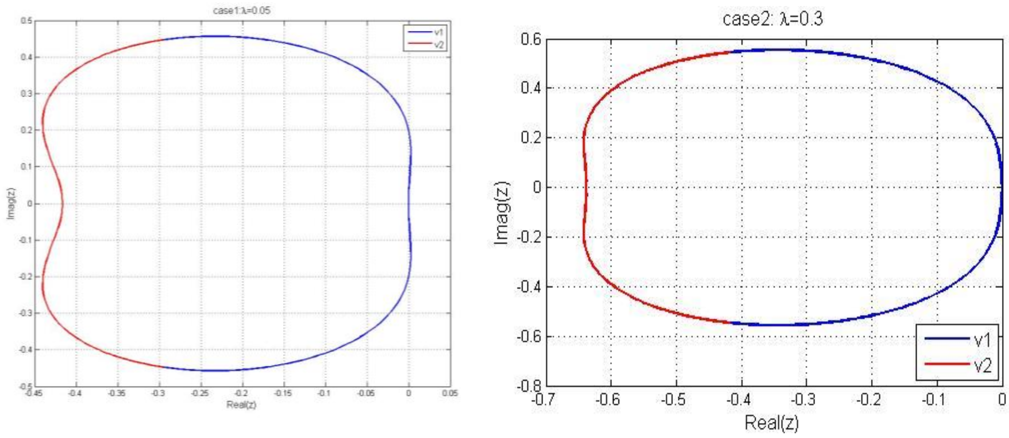


Fig. 2 Root locus for different parameters: (a) case 1 $\lambda = 0.05$, (b) case 2 $\lambda = 0.3$

4. CONCLUSION

The time delay is a critical parameter to the dynamical traffic flow. In this study, we propose an extended T-FVD-D model by taking into account two different time delays in sensing space headway and velocity. These two different time delays have important effects upon the property of the traffic flow, especially the stability condition. The stability analysis shows that the time delays would impact the stability of the traffic flow, while the stable region of the traffic flow model would increase with the deceasing of difference between two different time delays. Results from the simulations demonstrate the effect of the time delays to the novel traffic flow model.

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