

## Research on the Placement of Box Based on Mathematical Modeling

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*Abstract: Forklift truck transportation box problem is a common problem in life. It is a typical combinatorial optimization problem and has its own shape in various fields. In this paper, aiming at the problem of box placement, an optimization model is established by using the cyclic nesting method. Using LINGO and MATLAB to get the optimal placement scheme, and using Photoshop to draw the optimal scheme diagram. For question 1, the mathematical model of the maximum number of boxes that can be placed is established when the boxes are not allowed to exceed the bottom of forklift truck and can not overlap. We use the method of circular nesting to place boxes from outside to inside. Firstly, it is placed from the outermost layer to maximize the utilization rate of the boundary. Then, it fills the inner space until the box can not be placed. Using LINGO to solve the problem, the optimal layout scheme is obtained: the number of boxes can be 16 at most, the number of boxes can be 6 at most, and the number of boxes can be 20 at most. For question 2, the mathematical model of the maximum number of boxes that can be placed under the condition that the density of the boxes is uniform and the boxes are allowed to exceed the boards on the top, left and right sides of the square boards but cannot overlap with each other. A step-by-step optimization model from top to bottom is established to expand the length of half of the box to the left and right sides based on the bottom side. Place them from the bottom first, then stack them up so that there is no gap between the boxes, so as to maximize the area utilization rate until the upper side exceeds the length of half the boxes at most. By using LINGO, the optimal placement scheme is obtained: the number of boxes can be 23 at most, the number of boxes can be 8 at most, and the number of boxes can be 28 at most.*

*Keywords: Combinatorial optimization; circular nesting; maximum utilization.*

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### 1. RESTATEMENT OF THE PROBLEM

The problem of forklift packing is a very common problem in enterprises, and it is widely used in various fields such as production and transportation[1]. How to maximize the utilization of the forklift floor, thereby making full use of resources and saving space, is a concern of enterprises. The box is unified into a rectangle of the same shape and size, and the bottom plate of the forklift is a square with a side length of 1.1 meters[2-4]. Under the existing conditions, an optimized combination model is established to solve how to place the box, so that the floor area utilization is maximized. Question 1: If the box is not allowed to exceed the forklift floor (as shown in the above figure), and

the boxes are not allowed to overlap each other, establish an optimization model and consider how to place the boxes to maximize the number of boxes placed?

Question 2: Assume that the density of the box is uniform, allowing the box to be above the square bottom plate. The left and right sides are beyond the bottom plate (below the forklift wall and cannot be exceeded), but do not drop the forklift floor. In this case, re-establish the optimization model and calculate how many boxes can be placed at most? How to place it?

**2. PROBLEM ANALYSIS**

This paper studies the problem of stacking rectangular boxes on a square forklift floor with a side length of 1.1. The best placement of different sizes of boxes is different. It is necessary to meet the maximum number of boxes of different models. To design a universal best solution.

Problem 1 requires that the box not exceed the forklift floor and cannot overlap to consider the optimal solution. Firstly, referring to the loop nesting method, the MATLAB program that can maximize the area utilization is designed without the box exceeding the forklift floor. The LINGO solution is used to obtain the length-width combination and the number of small rectangles of the optimal solution. Then use the loop nesting method to get the optimal placement plan with the maximum utilization of the square side length, and draw the layout pattern with Photoshop[5].

Problem 2 Consider the optimal solution if the box can extend beyond the upper, left and right sides of the square base plate but cannot overlap. Because the density of the box is uniform, in order to maximize the area utilization, the left and right sides of the square bottom plate are firstly expanded by half the length of the box, and then the half length of the opposite upper side expanded box is used to obtain the maximum utilized rectangular area of the problem. Similar to the problem 1, the LINGO is used to obtain the optimum length-to-width ratio of the forklift wall, and the optimal scheme is obtained by stacking from the bottom to the top and then photographed by Photoshop[5].

**3. MODEL ASSUMPTIONS**

1 Assume that the density of the box is uniform. If the box is allowed to extend beyond the bottom plate on the upper, left and right sides of the forklift floor, as long as the center of gravity does not exceed the bottom plate.

2 The specifications of the box are uniform. When packing, it can be seen as a small rectangle, and the box can be placed tightly (no gap between the two boxes).

Table 1 Symbol Description

Symbol	Description
a	Small rectangular box
b	Small rectangular box width
c	The number of rectangular boxes superimposed on the long side
d	The number of rectangular boxes superimposed on the wide side
m	The number of long sides of the rectangular box on the bottom edge
n	The number of wide sides of the rectangular box on the bottom edge
sum	The total number of small rectangles

#### 4. MODEL ESTABLISHMENT AND SOLUTION

Question 1: Use the loop nesting method to fill from the outside to the inside. Place a small rectangle of length  $a$  and width  $b$  in a square with a side length of 1.1 to maximize the number of insertions. It is to use the various combinations of length and width to make the utilization of each side of the square the highest. After reasonable layout, the interior is filled to obtain the final layout pattern.

Establish the objective function that maximizes the length of the side:

$$\min z = 1.1 - a * m - b * n$$

The first type of box:  $a = 0.3$   $b = 0.24$   $l = 1.1$

Using LINGO to solve:  $m = 2$   $n = 2$

The layout is as follows:

The second box:  $a = 0.6$   $b = 0.4$   $l = 1.1$

Using LINGO to solve:  $m = 1$   $n = 1$

The layout is as follows:

The third box:  $a = 0.3$   $b = 0.2$   $l = 1.1$

Using LINGO to solve:  $m = 1$   $n = 4$

Question 2: Optimize the model step by step from bottom to top

1 Allow the box to be above the square bottom plate. The left and right sides are beyond the bottom plate (below the forklift wall and cannot be exceeded). In order to maximize the area, the long half of the box is expanded to the left and right of the square bottom plate to make it rectangular.

2 the bottom of the square bottom plate can not be exceeded, based on the bottom edge, first placed from the bottom edge, and then stacked up to make no gap between the boxes, so that the area utilization is maximum, up to the length of the upper half of the box.

Optimization goal: the largest number of small rectangles in the bottom plate.

Condition 1: The lower side expands the long side length  $a$  of the small rectangle, and the actual composition length is greater than  $l + a - b$  ,, less than or equal to  $l + a$ .

Condition 2: The length of the upward expansion must not be too small, nor can it be greater than half of the long side.

Condition 3: The width of the upward expansion should not be too small, nor should it be greater than half of the wide side.

Referring to the above design LINGO program, the objective function is:

$$\max f = m * c + n * d$$

The number  $m, n$  of the long side and the wide side of the small rectangle near the forklift wall and the numbers  $c$  and  $d$  accumulated by the long side and the wide side are obtained. The upper part is further filled with the bottom side as a base, so that there is no gap between the boxes, so that the area utilization rate is maximized.

Model solving:

The first type of box:  $a = 0.6$   $b = 0.24$   $l = 1.1$

Using LINGO to solve:  $m = 3$   $n = 2$   $c = 3$   $d = 4$

The layout is as follows:

The second box:  $a = 0.6$   $b = 0.4$   $l = 1.1$

Using LINGO to solve:  $m = 2$   $n = 1$   $c = 3$   $d = 2$

The layout is as follows:

The third box:  $a = 0.3$   $b = 0.2$   $l = 1.1$

Using LINGO to solve:  $m = 4$   $n = 1$   $c = 6$   $d = 4$

## 5. MODEL RESULTS ANALYSIS AND TESTING

For model 1:

The total area of the chassis is  $S$ , and the length and width of the small rectangle are  $a$ ,  $b$ , and  $N$  is the number of small rectangles when the chassis area utilization is maximum. It is obtained by  $N = S/(ab)$ :

$$N1 = 16, N2 = 5, N3 = 20, \text{sum1} = 16, \text{sum2} = 4, \text{sum3} = 20,$$

and the area utilization rate obtained by the model one is high, and the result is reasonable. In an ideal situation, the second model can be placed in five boxes, but according to the actual situation, the box shape is fixed and cannot be divided, so only four can be placed. So the result we get is the global optimal solution.

For model 2:

Because the condition allows the box to be above the square bottom plate, the left and right sides are beyond the bottom plate. As long as the center of gravity does not fall out of the bottom plate, it will not fall out of the forklift floor. Therefore, we assume that the other three sides expand outwardly by half of the long side, and the newly constructed rectangular bottom plate is tested as follows:

$$N1 = 23, N2 = 9, N3 = 29,$$

The model yields:

$$\text{sum1} = 23, \text{sum2} = 8, \text{sum3} = 28.$$

According to the actual situation, the shape of the box is fixed and cannot be divided, and in reality, not all the boxes are half out of the long side, so we get a better solution.

## REFERENCES

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